CLAIMS

1. A method for designing a non-linear system for transferring energy from a time or spatial domain input signal having a first spectrum at a first pre-determinable frequency or range of frequencies to a time or spatial domain output signal having a second spectrum at a second pre-determinable frequency or range of frequencies, said method comprising the steps of

identifying or specifying the first spectrum of the time or spatial domain input signal from which energy is to be transferred,

specifying the second spectrum of the time or spatial domain output signal to which said energy is to be transferred, and

calculating, using a frequency domain description of said output signal, for example, the output spectrum, expressed in terms of a frequency domain description of said input signal and coefficients of a time or spatial domain description of a generalised non-linear system, the coefficients of the time or spatial domain description of the generalised non-linear system in order to give effect to the energy transfer.

2. A method as claimed in claim 1, further comprising the step of

selecting a time or spatial domain description of the generalised non-linear system;

determining or defining a frequency domain description of the time or spatial domain input for the generalised non-linear system; and

determining or defining the frequency domain description of the output signal, for example, the output spectrum, of the generalised non-linear system expressed in terms of the frequency domain description of said input signal and the coefficients of the time or spatial domain description of a generalised non-linear system.

3. A method as claimed in either of claims 1 or 2, wherein the frequency domain description of the input signal is $U(jw)$, the time or spatial domain description of said generalised non-linear system is given by the generalised NARX model

$$y(k)=\sum_{n=1}^{N} y_n(k) \tag{C1}$$

where

$$y_n(k)=\sum_{p=0}^{n}\sum_{l_1,l_{p+q}=1}^{K_n} c_{pq}(l_1,\ldots,l_{p+q})\prod_{i=1}^{p} y(k-l_i)\prod_{i=p+1}^{p+q} u(k-l_i) \tag{C2}$$

with

$$p+q=n, \quad l_i=1,\ldots,K_n, \; i=1,\ldots,p+q, \text{ and } \sum_{l_1,l_{p+q}=1}^{K_n} \equiv \sum_{l_1=1}^{K_n}\cdots\sum_{l_{p+q}=1}^{K_n}$$

the frequency domain description of the output of the generalised non-linear system is given by

$$Y(jw)=\sum_{n=1}^{\overline{N}} Y_n(jw) \tag{C3}$$

where

$$Y_n(jw)=\frac{1/\sqrt{n}}{(2\pi)^{n-1}}\int_{w_1+\ldots+w_n=w} H_n(jw_1,\ldots,jw_n)\prod_{i=1}^{n} U(jw_i)d\sigma_w \tag{C4}$$

$\overline{N}$ is the maximum order of dominant system nonlinearities,

$$\int_{w_1+\ldots+w_n=w}(.)\,d\sigma_w$$

denotes an integration over the nth-dimensional hyper-plane $w_1+,\cdots,+w_n=w$, and $H_n(jw_1,\cdots,jw_n)$, $n=1,\ldots,\bar{N}$ are generalised frequency response functions of the non-linear system.

4. A method as claimed in any preceding claim, further comprising the step of

determining a mapping between the time or spatial domain description of the generalised nonlinear system and the frequency domain description of the generalised nonlinear system.

5. A method as claimed in claim 4, wherein the mapping from the time or spatial domain description of the generalised non-linear system to the frequency domain description of the system is given as

$$\begin{aligned}
&\left\{1-\sum_{l_1=1}^{K_1} c_{10}(l_1)\exp\left[-j\,(w_1+,\cdots,+w_n)\,l_1\right]\right\}H_n(jw_1,\cdots,jw_n) \\
&=\sum_{l_1,l_n=1}^{K_n} c_{0n}(l_1,\cdots,l_n)\exp\left[-j(w_1l_1+,\cdots,+w_nl_n)\right] \\
&+\sum_{q=1}^{n-1}\sum_{p=1}^{n-q}\sum_{l_1,l_{p+q}=1}^{K_{p+q}} c_{pq}(l_1,\cdots,l_{p+q})\exp\left[-j\left(w_{n-q+1}l_{p+1}+,\cdots,+w_nl_{p+q}\right)\right]H_{n-q,p}\left(jw_1,\cdots,jw_{n-q}\right) \\
&+\sum_{p=2}^{n}\sum_{l_1,l_p=1}^{K_p} c_{p0}(l_1,\cdots,l_p)H_{n,p}(jw_1,\cdots,jw_n)
\end{aligned} \tag{C5}$$

where

$$H_{np}(jw_1,\cdots,jw_n)=\sum_{i=1}^{n-p+1}H_i(jw_1,\cdots,jw_i)\,H_{n-i,p-1}(jw_{i+1},\cdots,jw_n)\exp\left[-j\,(w_1+,\cdots,+w_i)\,l_p\right] \tag{C6}$$

6. A method as claimed in any preceding claim, further comprising the steps of

defining or determining a general relationship between the input and output frequency or frequency ranges of the generalised non-linear system.

7. A method as claimed in any preceding claim, wherein the relationship between the input and output frequencies or frequency ranges is given by the following

$$f_Y = f_{Y_{\bar{N}}} \bigcup f_{Y_{\bar{N}-1}} \qquad \text{(C7)}$$

where f_Y denotes the range of frequencies of the output, and $f_{Y_{\bar{N}}}$ and $f_{Y_{\bar{N}-1}}$ denote the ranges of frequencies produced by the $\bar{N}$th-order and $(\bar{N}-1)$th-order nonlinearities, and

$$f_{Y_n} = \begin{cases} \bigcup_{k=0}^{i^*-1} I_k & \text{when } \frac{nb}{(a+b)} - \left[\frac{na}{(a+b)}\right] < 1 \\ \bigcup_{k=0}^{i^*} I_k & \text{when } \frac{nb}{(a+b)} - \left[\frac{na}{(a+b)}\right] \geq 1 \end{cases} \qquad \text{(C8)}$$

$$n = \bar{N} \quad \text{and} \quad \bar{N}-1$$

where [.] relates to or means take the integer part,

$$i^* = \left[\frac{na}{(a+b)}\right] + 1$$

$$I_k = [na - k(a+b), nb - k(a+b)] \quad \text{for } k = 0, \ldots, i^* - 1,$$

$$I_{i^*} = [0, nb - i^*(a+b)],$$

and the frequencies of the signal to be processed are in the range [a,b] and given [a,b] and the required output frequency range f_Y, the method further comprises the step of determining the smallest $\bar{N}$ from the relationship above for the generalised non-linear

system which can bring about the specified frequency domain energy transformation.

8. A method as claimed in claim 7, wherein, with $\overline{N}$ having been determined and $K_n, n=1,\ldots,\overline{N}$, being given a priori, the method further comprises the steps of:

taking N as $\overline{N}$ and determining the coefficients of the time or spatial domain model of the generalised non-linear system from the description for the system output spectrum given in terms of the spectrum of the input signal and the coefficients of the time or spatial domain model of the said generalised non-linear system.

9. A method as claimed in claim 8, further comprising the steps of

substituting $H_n(jw_1,\cdots,jw_n)$ given in (C5) into (C4), and substituting the resultant expression for $Y_n(jw)$ into (C3) to obtain the description for the system output spectrum in terms of a function of the spectrum of the input signal and the coefficients of the time or spatial domain model of the said generalised nonlinear system.

10. A method for realising or manufacturing a non-linear system for transferring energy from a time or spatial domain input signal having a first spectrum at a first pre-determinable frequency or range of frequencies to a time or spatial domain output signal having a second spectrum at a second pre-determinable frequency or range of frequencies, the method comprising the steps of

(a) designing the non-linear system using the method as claimed in any of claims 1 to 9; and

(b) materially producing the non-linear system so designed or using the non-linear system so designed to modify materially the transfer function of an existing linear or non-linear system.

11. A data processing system for designing a non-linear system for transferring energy from a time or spatial domain input signal having a first spectrum at a first pre-determinable frequency or range of frequencies to a time or spatial domain output signal having a second spectrum at a second pre-determinable frequency or range of frequencies, said system comprising

means for identifying or specifying the first spectrum of the time or spatial domain input signal from which energy is to be transferred,

means for specifying the second spectrum of the time or spatial domain output signal to which said energy is to be transferred, and

means for calculating, using a frequency domain description of said output signal, for example, the output spectrum, expressed in terms of a frequency domain description of said input and coefficients of a time or spatial domain description of a generalised non-linear system, the coefficients of the time or spatial domain description of said generalised non-linear system in order to give effect to the energy transfer.

12. A system as claimed in claim 11, further comprising

means for selecting a time or spatial domain description of the generalised non-linear system;

means for determining or defining a frequency domain description of the time or spatial domain input for the generalised non-linear system; and

means for determining or defining the frequency domain description of the output of the generalised non-linear system expressed in terms of the frequency domain description of said input signal and the coefficients of the time or spatial domain description of a generalised non-linear system.

13. A system as claimed in either of claims 11 or 12, wherein the frequency domain description of the input signal is $U(jw)$, the time or spatial domain description of the generalised non-linear system is given by the generalised NARX model

$$y(k)=\sum_{n=1}^{N} y_n(k) \qquad \text{(C9)}$$

where

$$y_n(k)=\sum_{p=0}^{n}\sum_{l_1,l_{p+q}=1}^{K_n} c_{pq}(l_1,\ldots,l_{p+q})\prod_{i=1}^{p} y(k-l_i)\prod_{i=p+1}^{p+q} u(k-l_i) \qquad \text{(C10)}$$

with

$$p+q=n, \qquad l_i=1,\ldots,K_n,\ i=1,\ldots,p+q,\ \text{and}\ \sum_{l_1,l_{p+q}=1}^{K_n}\equiv\sum_{l_1=1}^{K_n}\cdots\sum_{l_{p+q}=1}^{K_n}$$

the frequency domain description of the output of the generalised non-linear system is given by

$$Y(jw)=\sum_{n=1}^{\bar{N}} Y_n(jw) \qquad \text{(C11)}$$

where

$$Y_n(jw) = \frac{1/\sqrt{n}}{(2\pi)^{n-1}} \int_{w_1+,\ldots,+w_n=w} H_n(jw_1,\ldots,jw_n) \prod_{i=1}^{n} U(jw_i) d\sigma_w \qquad \text{(C12)}$$

$\overline{N}$ is the maximum order of dominant system nonlinearities,

$$\int_{w_1+,\ldots,+w_n=w} (.)\, d\sigma_w$$

denotes an integration over the nth-dimensional hyper-plane $w_1+,\cdots,+w_n = w$, and $H_n(jw_1,\cdots,jw_n)$, $n=1,\ldots,\overline{N}$ are generalised frequency response functions of the said non-linear system.

14. A system as claimed in any of claims 11 to 13, further comprising

means for determining a mapping between the time or spatial domain description of the generalised nonlinear system and the frequency domain description of the generalised nonlinear system.

15. A system as claimed in claim 14, wherein the mapping from the time or spatial domain description of the generalised non-linear system to the frequency domain description of the system is given as

$$\left\{1 - \sum_{l_1=1}^{K_1} c_{10}(l_1) \exp\left[-j\,(w_1+,\cdots,+w_n)\,l_1\right]\right\} H_n(jw_1,\cdots,jw_n)$$
$$= \sum_{l_1,l_n=1}^{K_n} c_{0n}(l_1,\cdots,l_n) \exp\left[-j(w_1 l_1+,\cdots,+w_n l_n)\right]$$
$$+ \sum_{q=1}^{n-1} \sum_{p=1}^{n-q} \sum_{l_1,l_{p+q}=1}^{K_{p+q}} c_{pq}(l_1,\cdots,l_{p+q}) \exp\left[-j(w_{n-q+1} l_{p+1}+,\cdots,+w_n l_{p+q})\right] H_{n-q,p}(jw_1,\cdots,jw_{n-q})$$
$$+ \sum_{p=2}^{n} \sum_{l_1,l_p=1}^{K_p} c_{p0}(l_1,\cdots,l_p) H_{n,p}(jw_1,\cdots,jw_n) \qquad \text{(C13)}$$

where

$$H_{np}(jw_1,\cdots,jw_n)=\sum_{i=1}^{n-p+1} H_i(jw_1,\cdots,jw_i)\,H_{n-i,p-1}(jw_{i+1},\cdots,jw_n)\exp\left[-j(w_1+\cdots+w_i)l_p\right] \tag{C14}$$

16. A system as claimed in any of claims 11 to 15, further comprising

means for defining or determining a general relationship between the input and output frequency or frequency ranges of the generalised non-linear system.

17. A system as claimed in any of claims 11 to 16, wherein the relationship between the input and output frequencies or frequency ranges is given by the following

$$f_Y = f_{Y_{\bar N}} \bigcup f_{Y_{\bar N-1}} \tag{C15}$$

where f_Y denotes the range of frequencies of the output, and $f_{Y_{\bar N}}$ and $f_{Y_{\bar N-1}}$ denote the ranges of frequencies produced by the $\bar N$th-order and $(\bar N-1)$th-order nonlinearities, and

$$f_{Y_n} = \begin{cases} \bigcup_{k=0}^{i^*-1} I_k & \text{when } \dfrac{nb}{(a+b)} - \left[\dfrac{na}{(a+b)}\right] < 1 \\ \bigcup_{k=0}^{i^*} I_k & \text{when } \dfrac{nb}{(a+b)} - \left[\dfrac{na}{(a+b)}\right] \geq 1 \end{cases} \tag{C16}$$

$$n = \bar N \quad \text{and} \quad \bar N - 1$$

where $[.]$ relates to or means take the integer part,

$$i^* = \left[\frac{na}{(a+b)}\right] + 1$$

$$I_k = [na - k(a+b),\, nb - k(a+b)] \quad \text{for } k = 0,\ldots,i^*-1,$$

$$I_{i^*} = [0,\, nb - i^*(a+b)],$$

and the frequencies of the signal to be processed are in the range $[a,b]$ and given $[a,b]$ and the required output

frequency range f_Y, the system further comprises the means for determining the smallest $\bar{N}$ from the relationship above for the said generalised non-linear system which can bring about the specified frequency domain energy transformation.

18. A system as claimed in claim 17, wherein, with $\bar{N}$ having been determined and $K_n, n=1,\ldots,\bar{N}$, being given a priori, the system further comprises the means:

for taking N as $\bar{N}$ and for determining the coefficients of the time or spatial domain model of the generalised non-linear system from the description for the system output spectrum given in terms of the spectrum of the input signal and the coefficients of the time or spatial domain model of the generalised non-linear system.

19. A system as claimed in claim 18, further comprising means for substituting $H_n(jw_1,\cdots,jw_n)$ given in (C13) into (C12), and substituting the resultant expression for $Y_n(jw)$ into (C11) to obtain the description for the system output spectrum in terms of a function of the spectrum of the input signal and the coefficients of the time or spatial domain model of the generalised nonlinear system.

20. A computer program product for designing a non-linear system for transferring energy from a time or spatial domain input signal having a first spectrum at a first pre-determinable frequency or range of frequencies to a time or spatial domain output signal having a second spectrum at a second pre-determinable frequency or range of frequencies, the said product comprising a computer readable storage medium comprising:

computer program code means for identifying or specifying the first spectrum of a time or spatial domain input signal from which energy is to be transferred,

computer program code means for specifying the second spectrum of a time or spatial domain output signal to which said energy is to be transferred, and

computer program code means for calculating, using a frequency domain description of the output signal, for example, the output spectrum, expressed in terms of a frequency domain description of the input and coefficients of a time or spatial domain description of a generalised non-linear system, the coefficients of a time or spatial domain description of said generalised non-linear system in order to give effect to the energy transfer.

21. A computer program product as claimed in claim 20, further comprising

computer program code means for selecting a time or spatial domain description of the generalised non-linear system;

computer program code means for determining or defining a frequency domain description of the time or spatial domain input for the generalised non-linear system; and

computer program code means for determining or defining the frequency domain description of the output of the generalised non-linear system expressed in terms of the frequency domain description of said input signal and the coefficients of the time or spatial domain description of a generalised non-linear system.

22. A computer program product as claimed in either of claims 20 or 21, wherein the frequency domain description of the input signal is *U(jw)*, the time or spatial domain description of the generalised non-linear system is given by the generalised NARX model

$$y(k)=\sum_{n=1}^{N} y_n(k) \tag{C17}$$

where

$$y_n(k)=\sum_{p=0}^{n}\sum_{l_1,l_{p+q}=1}^{K_n} c_{pq}(l_1,\ldots,l_{p+q})\prod_{i=1}^{p} y(k-l_i)\prod_{i=p+1}^{p+q} u(k-l_i) \tag{C18}$$

with

$$p+q=n, \quad l_i=1,\ldots,K_n, \quad i=1,\ldots,p+q, \text{ and } \sum_{l_1,l_{p+q}=1}^{K_n} \equiv \sum_{l_1=1}^{K_n}\cdots\sum_{l_{p+q}=1}^{K_n}$$

the frequency domain description of the output of the generalised non-linear system is given by

$$Y(jw)=\sum_{n=1}^{\bar{N}} Y_n(jw) \tag{C19}$$

where

$$Y_n(jw)=\frac{1/\sqrt{n}}{(2\pi)^{n-1}}\int_{w_1+,\ldots,+w_n=w} H_n(jw_1,\ldots,jw_n)\prod_{i=1}^{n} U(jw_i)d\sigma_w \tag{C20}$$

$\bar{N}$ is the maximum order of dominant system nonlinearities,

$$\int_{w_1+,\ldots,+w_n=w}(.)\,d\sigma_w$$

denotes an integration over the nth-dimensional hyper-plane $w_1+,\cdots,+w_n=w$, and $H_n(jw_1,\cdots,jw_n)$, $n=1,\ldots,\bar{N}$ are generalised frequency response functions of the non-linear system.

23. A computer program product as claimed in any of claims 20 to 22, further comprising

computer program code means for determining a mapping between the time or spatial domain description of the generalised nonlinear system and the frequency domain description of the generalised nonlinear system.

24. A computer program product as claimed in claim 23, wherein the mapping from the time or spatial domain description of the generalised non-linear system to the frequency domain description of the system is given as

$$\begin{aligned}
&\left\{1-\sum_{l_1=1}^{K_1} c_{10}(l_1)\exp\left[-j\,(w_1+,\cdots,+w_n)\,l_1\right]\right\}H_n(jw_1,\cdots,jw_n)\\
&=\sum_{l_1,l_n=1}^{K_n} c_{0n}(l_1,\cdots,l_n)\exp\left[-j(w_1 l_1+,\cdots,+w_n l_n)\right]\\
&+\sum_{q=1}^{n-1}\sum_{p=1}^{n-q}\sum_{l_1,l_{p+q}=1}^{K_{p+q}} c_{pq}(l_1,\cdots,l_{p+q})\exp\left[-j\left(w_{n-q+1}l_{p+1}+,\cdots,+w_n l_{p+q}\right)\right]H_{n-q,p}\left(jw_1,\cdots,jw_{n-q}\right)\\
&+\sum_{p=2}^{n}\sum_{l_1,l_p=1}^{K_p} c_{p0}\left(l_1,\cdots,l_p\right)H_{n,p}(jw_1,\cdots,jw_n)
\end{aligned} \tag{C21}$$

where

$$H_{np}(jw_1,\cdots,jw_n)=\sum_{i=1}^{n-p+1}H_i(jw_1,\cdots,jw_i)\,H_{n-i,p-1}(jw_{i+1},\cdots,jw_n)\exp\left[-j\,(w_1+,\cdots,+w_i)\,l_p\right] \tag{C22}$$

25. A computer program product as claimed in any of claims 20 to 24, further comprising

computer program code means for defining or determining a general relationship between the input and output frequency or frequency ranges of the generalised non-linear system.

26. A computer program product as claimed in any of claims 20 to 25, wherein the relationship between the input

and output frequencies or frequency ranges is given by the following

$$f_Y = f_{Y_{\bar{N}}} \bigcup f_{Y_{\bar{N}-1}} \qquad \text{(C23)}$$

where f_Y denotes the range of frequencies of the output, and $f_{Y_{\bar{N}}}$ and $f_{Y_{\bar{N}-1}}$ denotes the ranges of frequencies produced by the $\bar{N}$th-order and $(\bar{N}-1)$th-order nonlinearities, and

$$f_{Y_n} = \begin{cases} \bigcup\limits_{k=0}^{i^*-1} I_k & \text{when } \dfrac{nb}{(a+b)} - \left[\dfrac{na}{(a+b)} \right] < 1 \\ \bigcup\limits_{k=0}^{i^*} I_k & \text{when } \dfrac{nb}{(a+b)} - \left[\dfrac{na}{(a+b)} \right] \geq 1 \end{cases} \qquad \text{(C24)}$$

$$n = \bar{N} \quad \text{and} \quad \bar{N}-1$$

where [.] relates to or means take the integer part,

$$i^* = \left[\frac{na}{(a+b)} \right] + 1$$

$$I_k = [na - k(a+b), nb - k(a+b)] \quad \text{for } k = 0, \ldots, i^* - 1,$$

$$I_{i^*} = [0, nb - i^*(a+b)],$$

and the frequencies of the signal to be processed are in the range [a,b], given [a,b] and the required output frequency range f_Y, the computer program product further comprises computer program code means for determining the smallest $\bar{N}$ from the relationship above for the said generalised non-linear system which can bring about the specified frequency domain energy transformation.

27. A computer program product as claimed in claim 26, wherein, with $\bar{N}$ having been determined and $K_n, n=1,\ldots,\bar{N}$, being given a priori, said product further comprises computer program code means for taking N as $\bar{N}$ and determining the coefficients of the time or spatial domain model of the generalised non-linear system from

taking N as $\bar{N}$ and determining the coefficients of the time or spatial domain model of the generalised non-linear system from the description for the system output spectrum given in terms of the spectrum of the input signal and the coefficients of the time or spatial domain model of the said generalised non-linear system.

28. A computer program product as claimed in any of claims 20 to 27, further comprising computer program code means for substituting $H_n(jw_1,\cdots,jw_n)$ given in (C21) into (C20), and substituting the resultant expression for $Y_n(jw)$ into (C19) to obtain the description for the system output spectrum in terms of a function of the spectrum of the input signal and the coefficients of the time or spatial domain model of the generalised nonlinear system.

29. A non-linear system which can transfer energy from a time or spatial domain input signal having a first spectrum at a first pre-determinable frequency or range of frequencies to a time or spatial domain output signal having a second spectrum at a second pre-determinable frequency or range of frequencies, said system comprising

means for identifying the first spectrum of the time or spatial domain input signal from which energy is to be transferred,

means for specifying the second spectrum of the time or spatial domain output signal to which said energy is to be transferred, and

means for giving effect to the energy transfer using coefficients of a time or spatial domain description of a generalised non-linear system, said coefficients having been calculated using a frequency domain description of said output signal, for example, the output spectrum, expressed in terms of a frequency domain description of said input signal and coefficients of a time or spatial domain description of a generalised non-linear system.

30. A non-linear system to transferring energy from a time or spatial domain input signal having a first spectrum at a first pre-determinable frequency or range of frequencies to a time or spatial domain output signal having a second spectrum at a second pre-determinable frequency or range of frequencies, said system comprising means for implementing a method as claimed in any of claims 1 to 9.

31. A non-linear system for transferring energy from a time or spatial domain input signal having a first spectrum at a first pre-determinable frequency or range of frequencies to a time or spatial domain output signal having a second spectrum at a second pre-determinable frequency or range of frequencies, said non-linear system comprising a data processing system as claimed in any of claims 11 to 19.

32. An article of manufacture comprising a computer usable medium with computer readable program code means embodied in the medium for designing a non-linear system for transferring energy from a time or spatial domain input signal having a first spectrum at a first pre-determinable frequency or range of frequencies to a

time or spatial domain output signal having a second spectrum at a second pre-determinable frequency or range of frequencies, the computer readable program code means in said article comprising:

computer readable program code means for identifying the first spectrum of a time or spatial domain input signal from which energy is to be transferred,

computer readable program code means for specifying the second spectrum of a time or spatial domain output signal to which said energy is to be transferred, and

computer readable program code means for calculating, using a frequency domain description of the output signal, for example, the output spectrum, expressed in terms of a frequency domain description of the input and coefficients of a time or spatial domain description of a generalised non-linear system, the coefficients of a time or spatial domain description of said generalised non-linear system in order to give effect to the energy transfer.